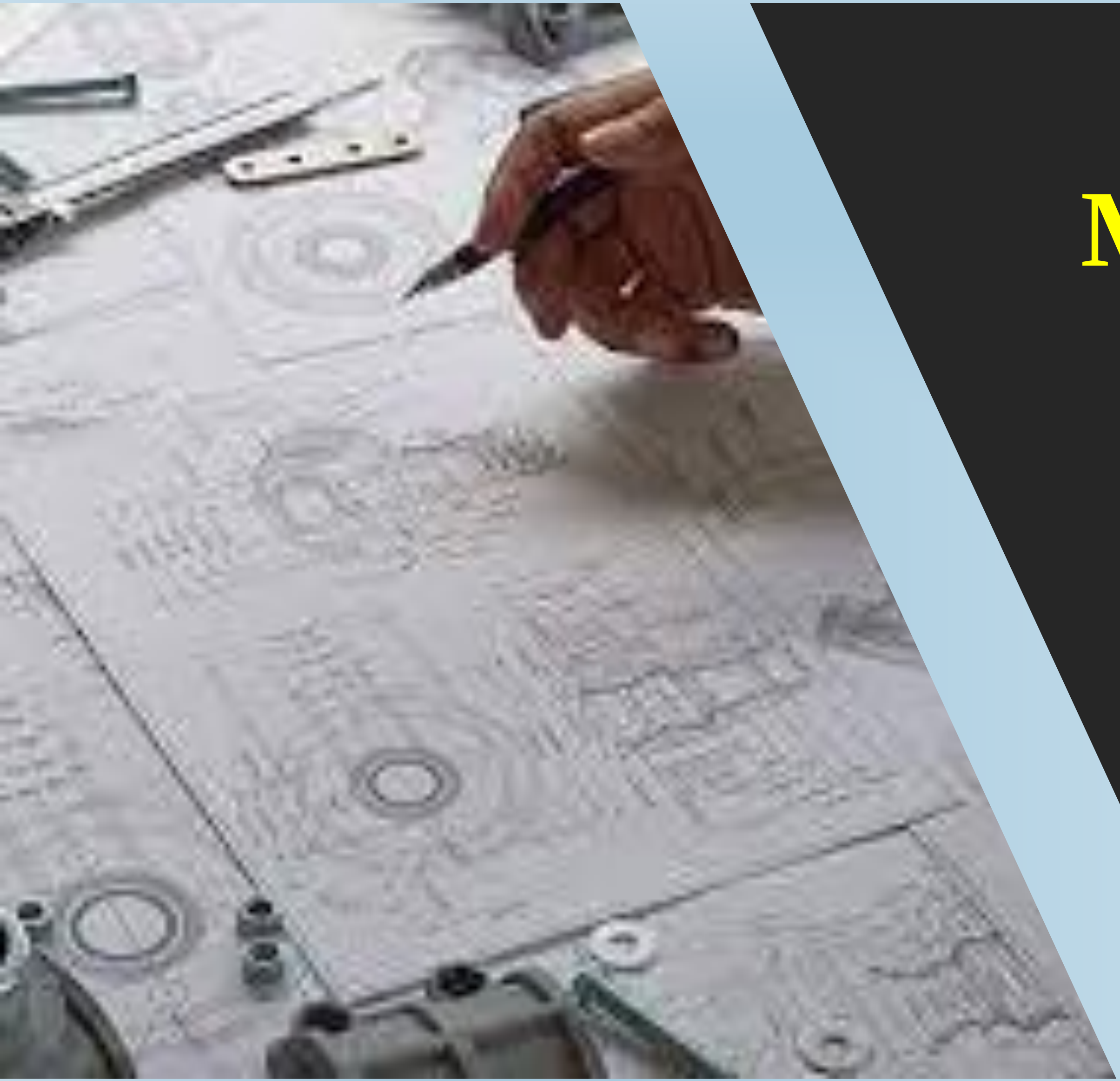
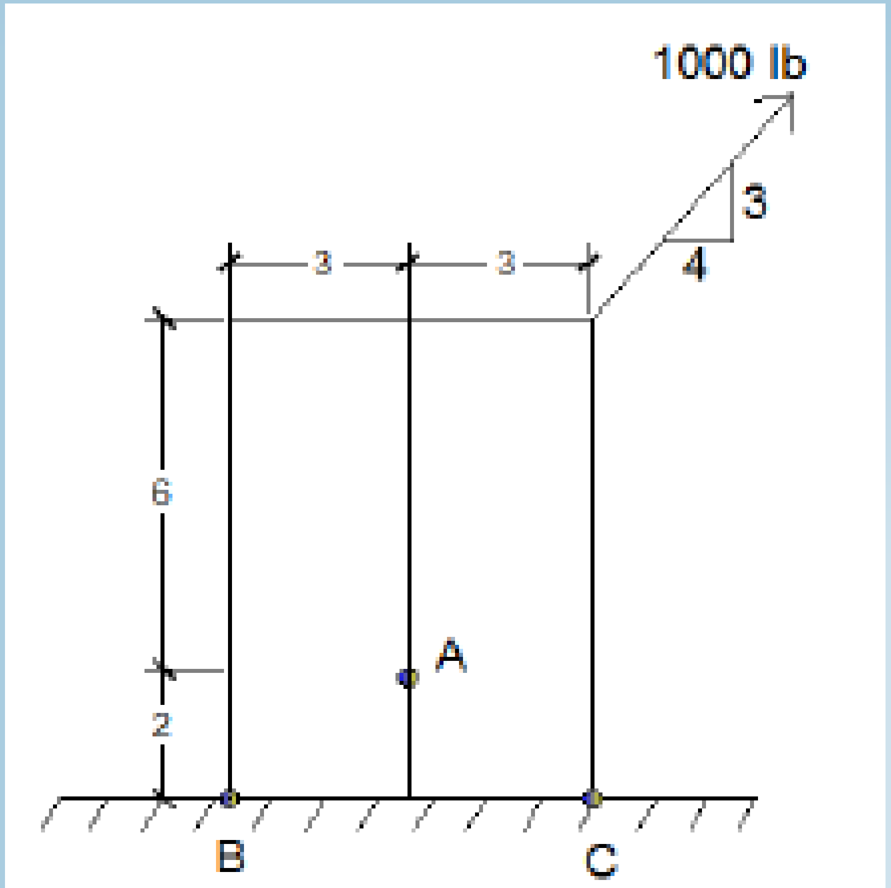


MECHANICS

Lecture No.4

د. محمد سعد

Example: Replace the force and couple in the fig. 4 couple whose forces are vertically at C and B.



SOL.

Move force 100 Ib to point A

$$Ma = 1000 * 6 * 0.8 - 1000 * 3 * 0.6 = 3000\ Ib.ft$$

Find the couple forces

$$M_{BC} = F * d$$

$$3000 = F * 6$$

$$F = 500\ Ib$$

Example A circular disc of radius 1 m is acted upon by four forces as shown in Fig. 4.33. Replace the forces by a single equivalent force.

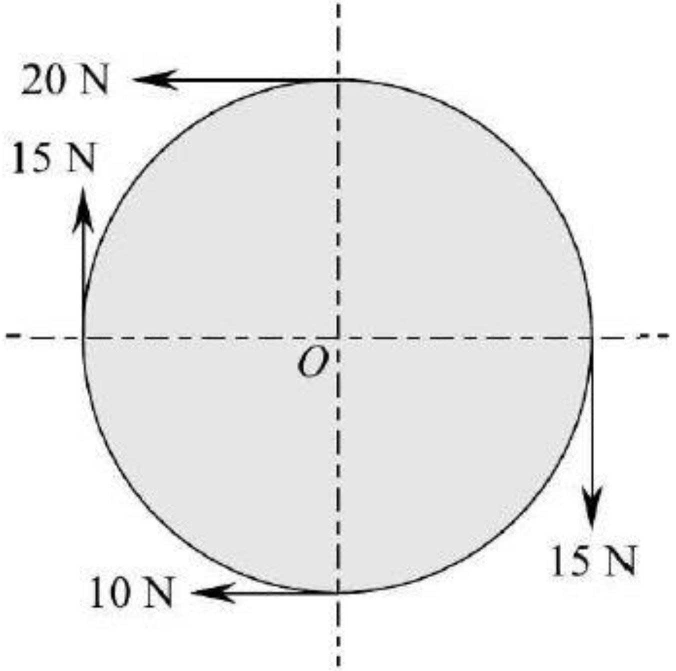


Fig. 4.33

$$\sum M_O = 20 \times 1 - 10 \times 1 - 15 \times 2 = -20 \text{ N.m}$$

$$\sum F_x = -20 - 10 = -30 \text{ N}$$

Fig. 4.33

As the two vertical forces are equal in magnitude and opposite to each other, they form a couple. Hence,

$$\sum F_y = 0$$

As $\sum F_y$ is zero, x-intercept does not exist. Therefore,

$$y = -\frac{\sum M_O}{\sum F_x}$$

$$= -\left[\frac{-20}{-30}\right] = -0.67 \text{ m}$$

The *equivalent force* is shown in Fig. 4.33(b).

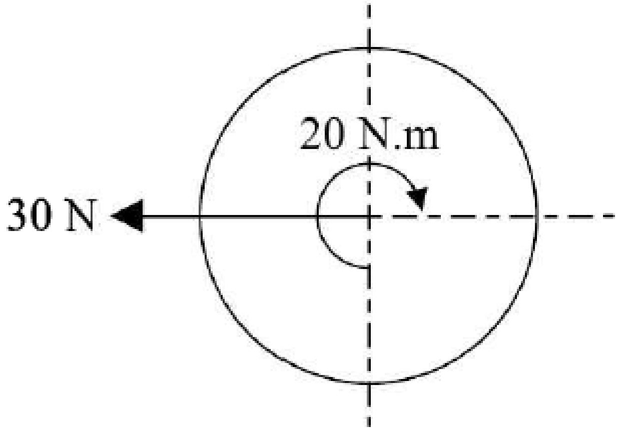


Fig. 4.33(a)

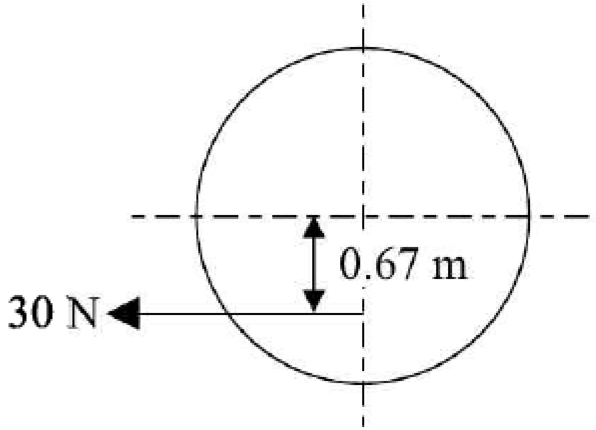
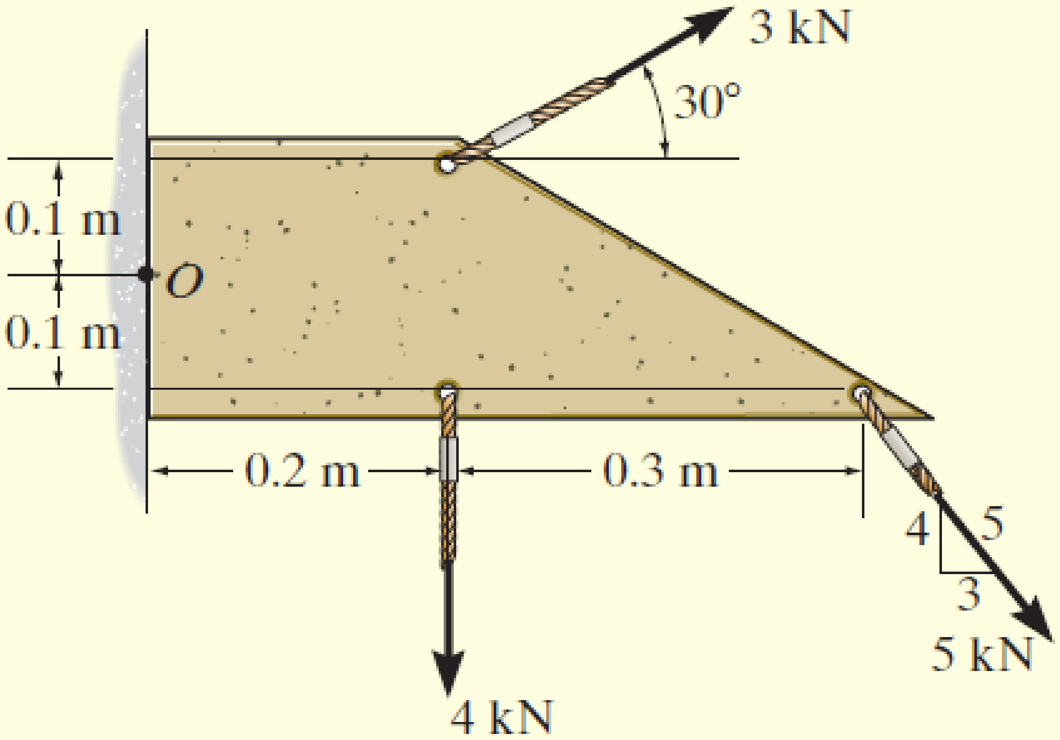


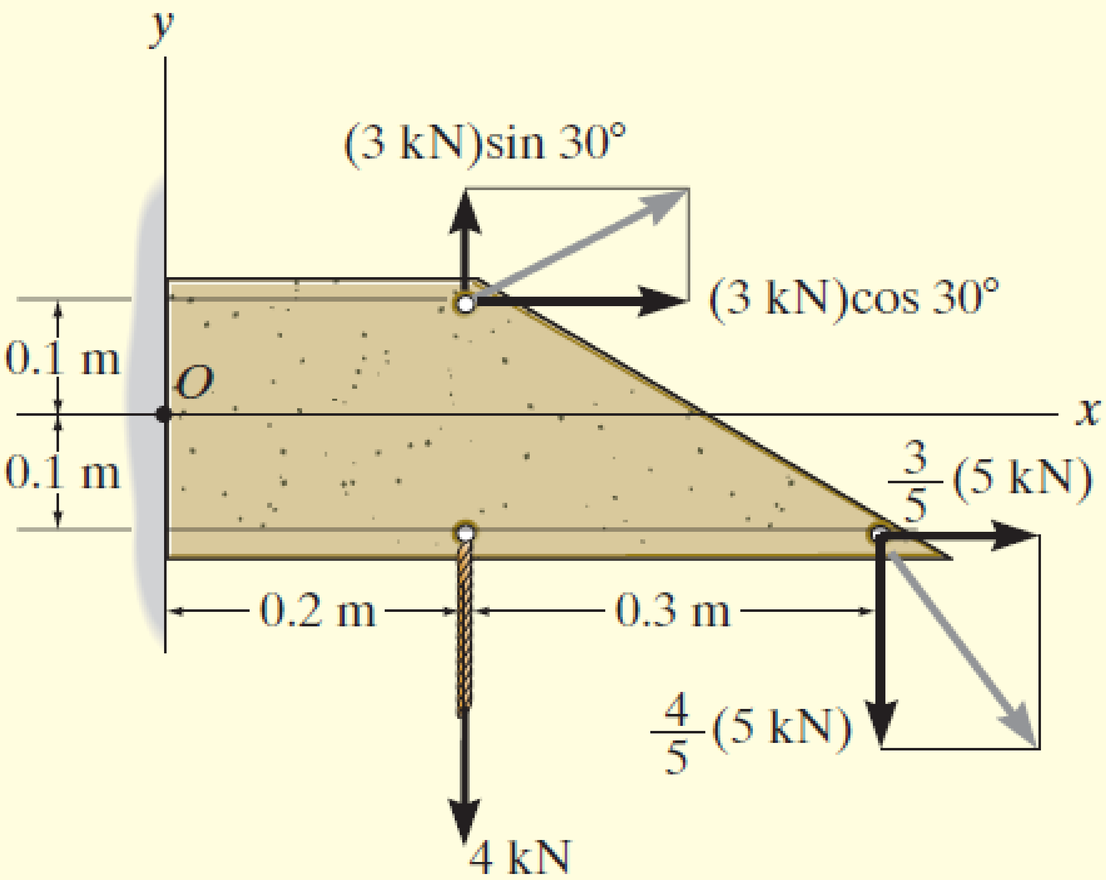
Fig. 4.33(b)

Example

Replace the force and couple system shown in Fig. 4–37*a* by an equivalent resultant force and couple moment acting at point *O*.



(a)



(b)

SOLUTION

Force Summation. The 3 kN and 5 kN forces are resolved into their x and y components as shown in Fig. 4–37b. We have

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = (3 \text{ kN}) \cos 30° + \left(\tfrac{3}{5}\right)(5 \text{ kN}) = 5.598 \text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = (3 \text{ kN}) \sin 30° - \left(\tfrac{4}{5}\right)(5 \text{ kN}) - 4 \text{ kN} = -6.50 \text{ kN} = 6.50 \text{ kN} \downarrow$$

Using the Pythagorean theorem, Fig. 4–37c, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{(5.598 \text{ kN})^2 + (6.50 \text{ kN})^2} = 8.58 \text{ kN} \qquad \textit{Ans.}$$

Its direction θ is

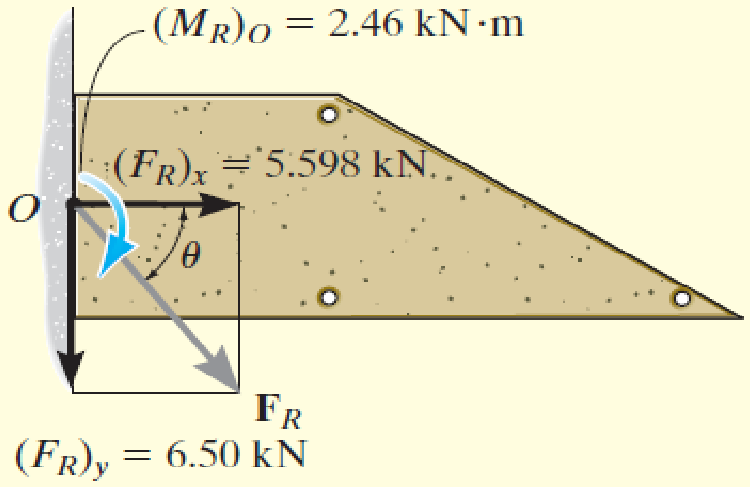
$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{6.50 \text{ kN}}{5.598 \text{ kN}}\right) = 49.3° \qquad \textit{Ans.}$$

Moment Summation. The moments of 3 kN and 5 kN about point O will be determined using their x and y components. Referring to Fig. 4–37b, we have

$$\circlearrowleft + (M_R)_O = \Sigma M_O;$$

$$(M_R)_O = (3 \text{ kN}) \sin 30°(0.2 \text{ m}) - (3 \text{ kN}) \cos 30°(0.1 \text{ m}) + \left(\tfrac{3}{5}\right)(5 \text{ kN})\,(0.1 \text{ m})$$
$$- \left(\tfrac{4}{5}\right)(5 \text{ kN})\,(0.5 \text{ m}) - (4 \text{ kN})(0.2 \text{ m})$$
$$= -2.46 \text{ kN}\cdot\text{m} = 2.46 \text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

This clockwise moment is shown in Fig. 4–37c.



(c)

Example

Replace the force and couple system acting on the member in Fig. 4–38*a* by an equivalent resultant force and couple moment acting at point *O*.

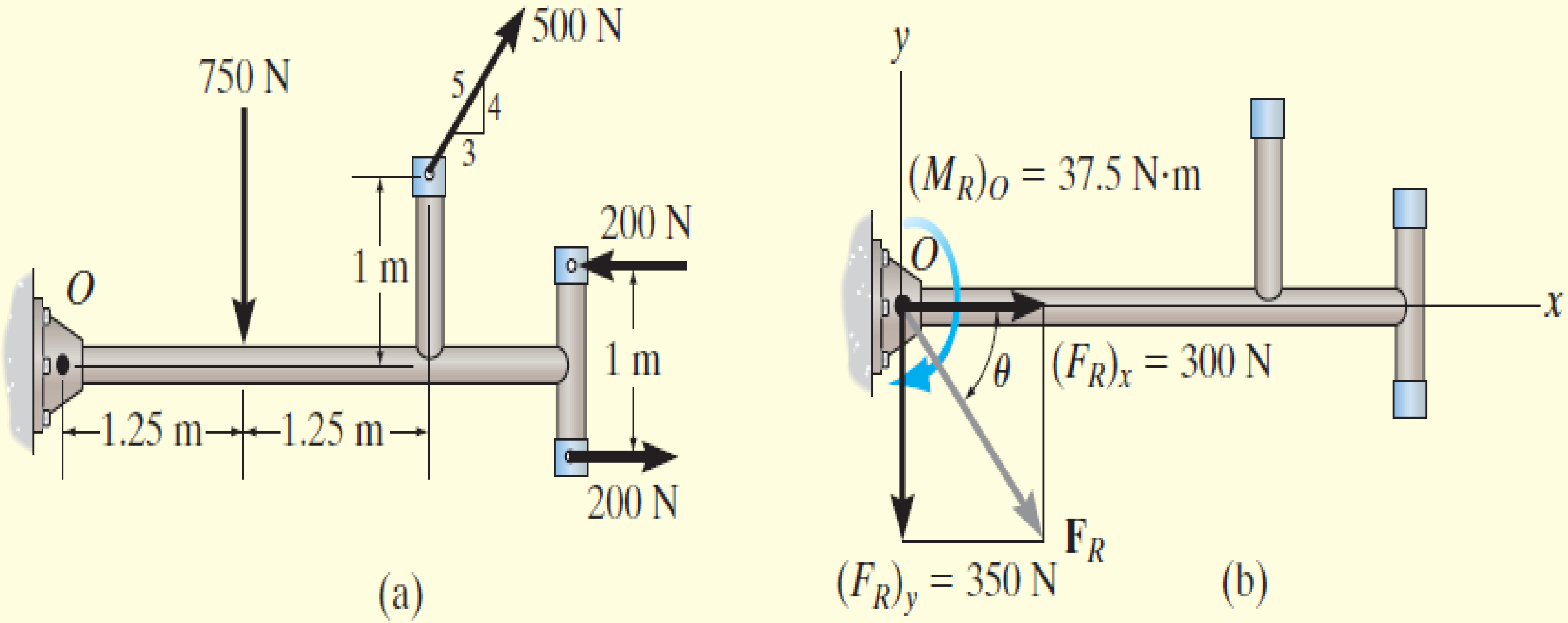


Fig. 4–38

SOLUTION

Force Summation. Since the couple forces of 200 N are equal but opposite, they produce a zero resultant force, and so it is not necessary to consider them in the force summation. The 500-N force is resolved into its x and y components, thus,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \quad (F_R)_x = \left(\tfrac{3}{5}\right)(500\text{ N}) = 300\text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = (500\text{ N})\left(\tfrac{4}{5}\right) - 750\text{ N} = -350\text{ N} = 350\text{ N}\downarrow$$

From Fig. 4–15b, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$= \sqrt{(300\text{ N})^2 + (350\text{ N})^2} = 461\text{ N} \qquad \textit{Ans.}$$

And the angle θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{350\text{ N}}{300\text{ N}}\right) = 49.4^\circ \qquad \textit{Ans.}$$

Moment Summation. Since the couple moment is a free vector, it can act at any point on the member. Referring to Fig. 4–38a, we have

$$\circlearrowleft + (M_R)_O = \Sigma M_O + \Sigma M$$

$$(M_R)_O = (500\text{ N})\left(\tfrac{4}{5}\right)(2.5\text{ m}) - (500\text{ N})\left(\tfrac{3}{5}\right)(1\text{ m})$$

$$- (750\text{ N})(1.25\text{ m}) + 200\text{ N}\cdot\text{m}$$

$$= -37.5\text{ N}\cdot\text{m} = 37.5\text{ N}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

This clockwise moment is shown in Fig. 4–38b.

Example Four forces and a couple are applied to a rectangular plate as shown in Fig. 4.26. Replace the forces and the couple by an equivalent force–couple system at the bolt 1.

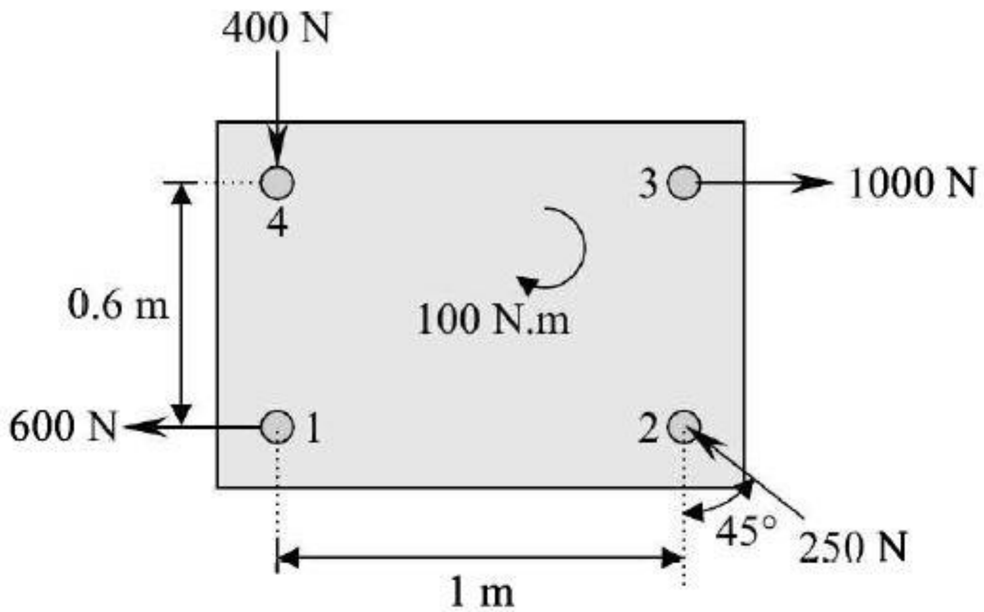


Fig. 4.26

Solution Taking summation of all the forces along X and Y directions,

$$\sum F_x = -600 - 250 \sin 45° + 1000$$
$$= 223.22 \text{ N}$$

$$\sum F_y = -400 + 250 \cos 45^\circ$$
$$= -223.22 \text{ N}$$

Therefore, the resultant force is obtained as

$$R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2} = 315.68 \text{ N}$$

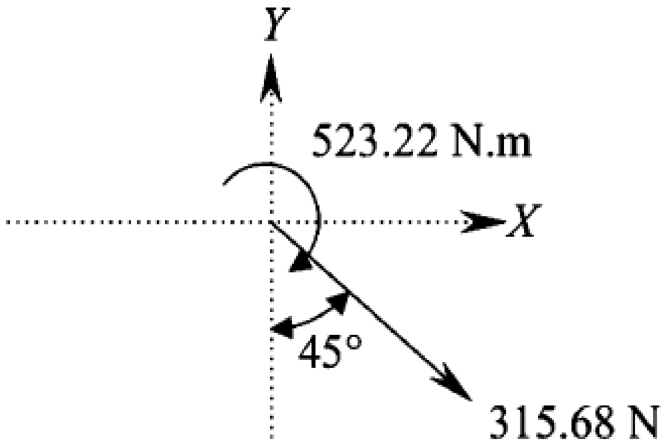


Fig. 4.26(a)

Its inclination with respect to the X-axis is obtained as

$$\theta = \tan^{-1}\left(\frac{\left|\sum F_y\right|}{\left|\sum F_x\right|}\right)$$
$$= \tan^{-1}\left(\frac{223.22}{223.22}\right) = 45^\circ$$

Since $\sum F_x$ is positive and $\sum F_y$ is negative, we know that the resultant lies in the fourth quadrant.

Taking summation of the moments of all the forces about the bolt 1,

$$\sum M_1 = -100 + (250 \cos 45^\circ \times 1) - (1000 \times 0.6)$$
$$= -523.22 \text{ N.m}$$

Example An equilateral triangular plate of side 3 m is acted on by three forces as shown in Fig. 4.27. Replace them by an equivalent force–couple system at *A*.

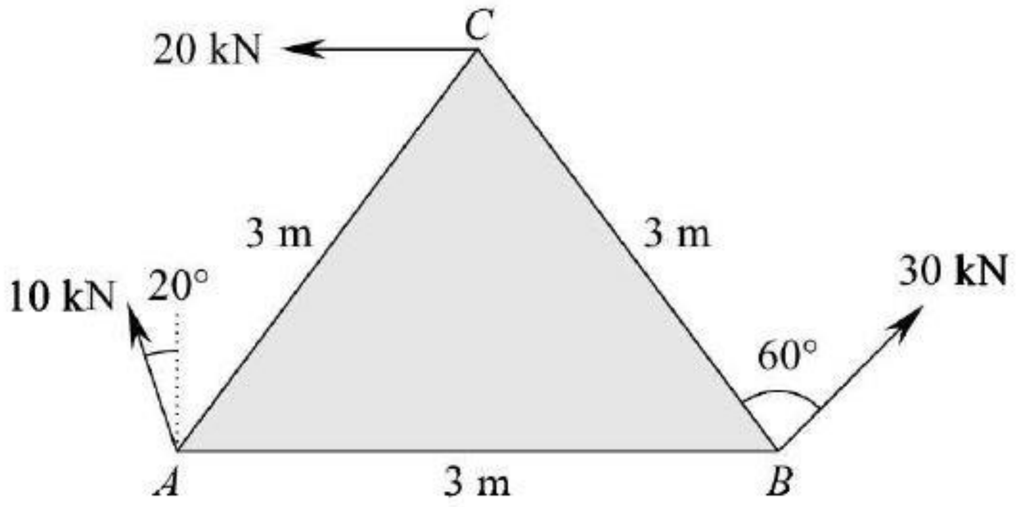


Fig. 4.27

Solution The given forces are resolved into rectangular components as shown in Fig. 4.27(a).

Taking summation of the forces along X and Y axes:

$$\sum F_x = -10 \sin 20° - 20 + 30 \cos 60°$$

$$= -8.42 \text{ kN}$$

$$\sum F_y = 10 \cos 20° + 30 \sin 60°$$

$$= 35.38 \text{ kN}$$

Therefore, the resultant force is obtained as

$$R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2} = 36.37 \text{ kN}$$

Its inclination with respect to the X-axis is

$$\theta = \tan^{-1}\left(\frac{\left|\sum F_y\right|}{\left|\sum F_x\right|}\right)$$

$$= \tan^{-1}\left(\frac{35.38}{8.42}\right) = 76.61°$$

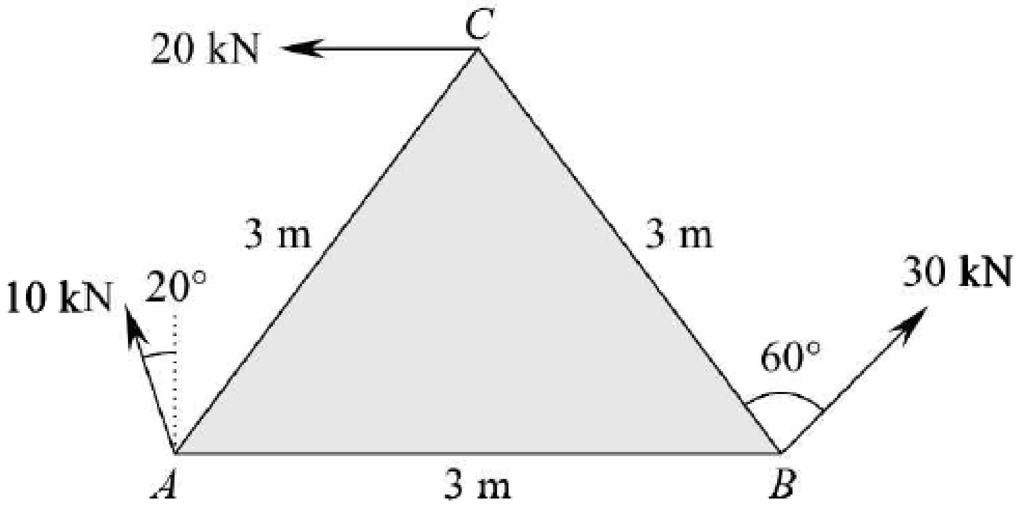


Fig. 4.27

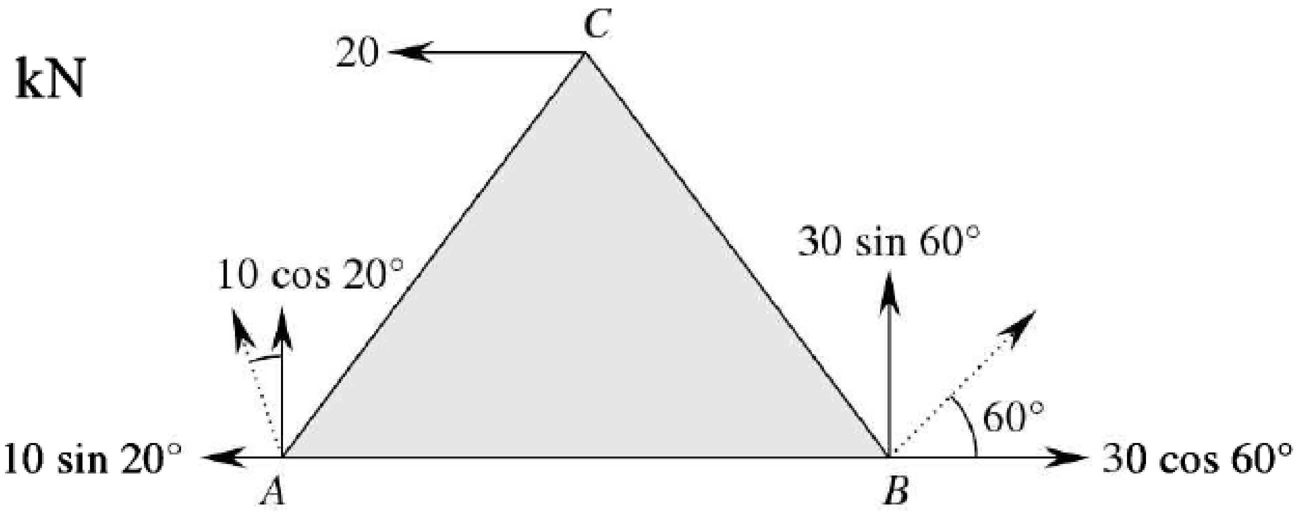


Fig. 4.27(a)

Since $\sum F_x$ is negative and $\sum F_y$ is positive, we know that the resultant lies in the second quadrant.

From Fig. 4.27(b), we know that

$$\overline{CD} = \sqrt{3^2 - (1.5)^2} = \sqrt{6.75} \text{ m}$$

Taking summation of the moments of all the forces about A,

$$M_A = 20 \times \sqrt{6.75} + 30 \sin 60° \times 3$$

$$= 129.9 \text{ N.m}$$

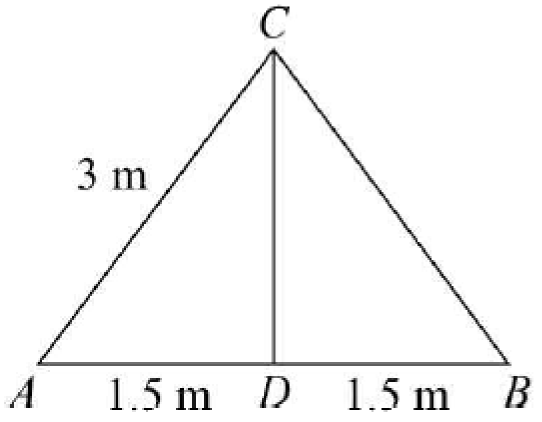


Fig. 4.27(b)

Replace the force and couple moment system acting on the beam in Fig. 4–44*a* by an equivalent resultant force, and find where its line of action intersects the beam, measured from point *O*.

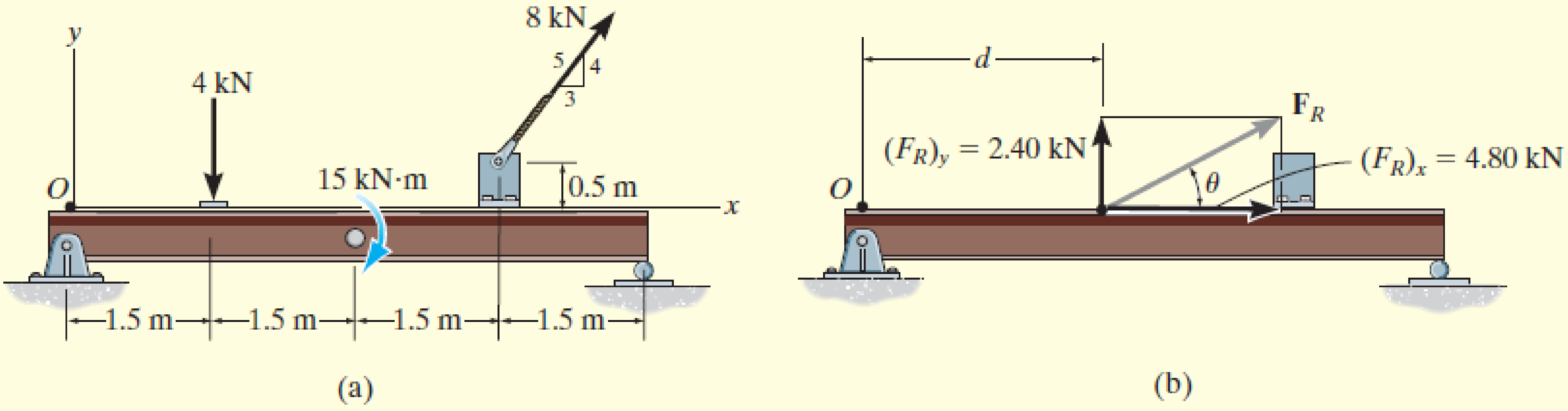


Fig. 4–44

SOLUTION

Force Summation. Summing the force components,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 8\text{ kN}\left(\tfrac{3}{5}\right) = 4.80\text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -4\text{ kN} + 8\text{ kN}\left(\tfrac{4}{5}\right) = 2.40\text{ kN}\uparrow$$

From Fig. 4–44*b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(4.80\text{ kN})^2 + (2.40\text{ kN})^2} = 5.37\text{ kN} \qquad \textit{Ans.}$$

The angle θ is

$$\theta = \tan^{-1}\left(\frac{2.40\text{ kN}}{4.80\text{ kN}}\right) = 26.6^\circ \qquad \textit{Ans.}$$

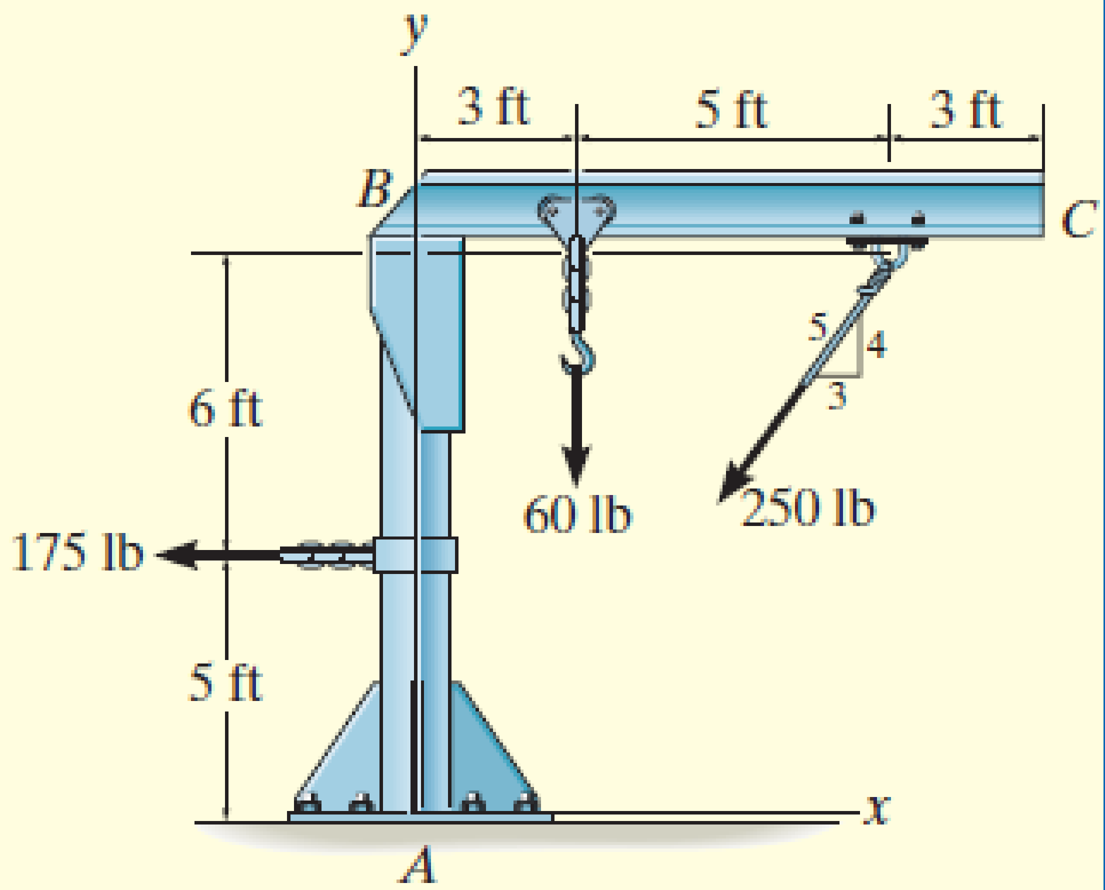
Moment Summation. We must equate the moment of $\mathbf{F}_R$ about point O in Fig. 4–44*b* to the sum of the moments of the force and couple moment system about point O in Fig. 4–44*a*. Since the line of action of $(\mathbf{F}_R)_x$ acts through point O, *only* $(\mathbf{F}_R)_y$ *produces a moment* about this point. Thus,

$$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad 2.40\text{ kN}(d) = -(4\text{ kN})(1.5\text{ m}) - 15\text{ kN}\cdot\text{m}$$

$$-\left[8\text{ kN}\left(\tfrac{3}{5}\right)\right](0.5\text{ m}) + \left[8\text{ kN}\left(\tfrac{4}{5}\right)\right](4.5\text{ m})$$

$$d = 2.25\text{ m} \qquad \textit{Ans.}$$

The jib crane shown in Fig. 4–45*a* is subjected to three coplanar forces. Replace this loading by an equivalent resultant force and specify where the resultant's line of action intersects the column *AB* and boom *BC*.



(a)

SOLUTION

Force Summation. Resolving the 250-lb force into x and y components and summing the force components yields

$$\overset{+}{\rightarrow}(F_R)_x = \Sigma F_x; \quad (F_R)_x = -250\text{ lb}\left(\tfrac{3}{5}\right) - 175\text{ lb} = -325\text{ lb} = 325\text{ lb} \leftarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \quad (F_R)_y = -250\text{ lb}\left(\tfrac{4}{5}\right) - 60\text{ lb} = -260\text{ lb} = 260\text{ lb}\downarrow$$

As shown by the vector addition in Fig. 4–45*b*,

$$F_R = \sqrt{(325\text{ lb})^2 + (260\text{ lb})^2} = 416\text{ lb} \qquad \textit{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{260\text{ lb}}{325\text{ lb}}\right) = 38.7° \qquad \textit{Ans.}$$

Moment Summation. Moments will be summed about point A. Assuming the line of action of $\mathbf{F}_R$ *intersects* AB at a distance y from A, Fig. 4–45*b*, we have

$$\zeta + (M_R)_A = \Sigma M_A; \qquad 325\text{ lb }(y) + 260\text{ lb }(0)$$

$$= 175\text{ lb }(5\text{ ft}) - 60\text{ lb }(3\text{ ft}) + 250\text{ lb}\left(\tfrac{3}{5}\right)(11\text{ ft}) - 250\text{ lb}\left(\tfrac{4}{5}\right)(8\text{ ft})$$

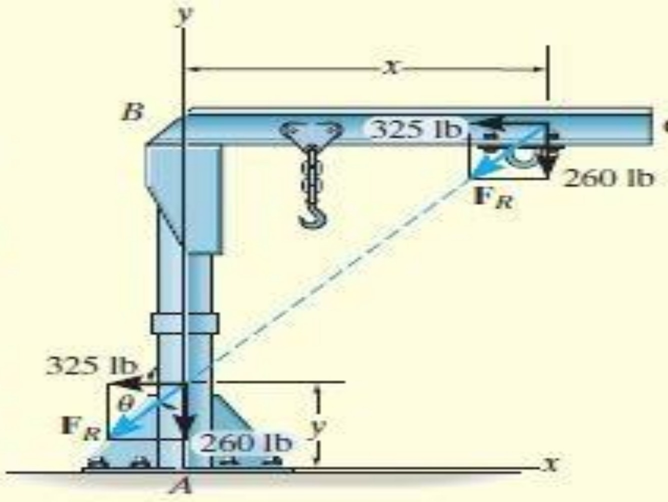
$$y = 2.29\text{ ft} \qquad \textit{Ans.}$$

By the principle of transmissibility, $\mathbf{F}_R$ can be placed at a distance x where it intersects BC, Fig. 4–45*b*. In this case we have

$$\zeta + (M_R)_A = \Sigma M_A; \qquad 325\text{ lb }(11\text{ ft}) - 260\text{ lb }(x)$$

$$= 175\text{ lb }(5\text{ ft}) - 60\text{ lb }(3\text{ ft}) + 250\text{ lb}\left(\tfrac{3}{5}\right)(11\text{ ft}) - 250\text{ lb}\left(\tfrac{4}{5}\right)(8\text{ ft})$$

$$x = 10.9\text{ ft} \qquad \textit{Ans.}$$



(b)

Fig. 4–45

Thank you for listening